Enrollment No: _______________________ **Exam Seat No:**_______________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Engineering Mathematics - II

Subject Code: 4TE02EMT3 **Branch: B.Tech (All)**

Semester: 2 **Date: 19/09/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\cdots+\infty$ is
(a) convergent (b) divergent (c) finitely oscillating (d) infinitely oscillating

b) Telescoping series is
(a) convergent (b) divergent (c) finitely oscillating (d) None of these

c) $\int_0^{\pi/2} sin^4\, x\, dx =$ ______.
$(a)\ \frac{3\pi}{16}$ $(b)\ \frac{6\pi}{16}$ $(c)\ \frac{9\pi}{16}$ $(d)\ \frac{3\pi}{4}$

d) If equation of the curve is $y=f(x)$, then length of the curve is ______
$(a)\ s=\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ $(b)\ s=\int_{x_1}^{x_2}\sqrt{1+\frac{dy}{dx}}\,dx$
$(c) s=\int_{x_1}^{x_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\,dx$ (d)None of these

e) $\Gamma(n-1) =$ _____. $(n \geq 2)$
$(a)\ (n-2)!$ $(b)\ (n-3)!$ $(c)\ (n-4)!$ $(d)\ (n-5)!$

f) $\beta\left(\frac{1}{2},\frac{1}{2}\right) =$ _____.
$(a)\ \sqrt{\pi}$ $(b)\ 1$ $(c)\ 0$ $(d)\ \pi$

g) For $m > o$, Gamma Function is_____
(a) convergent (b) divergent (c) finitely oscillating (d) None of these

h) Area of Region R is ______

(a) $\int_R x\,dy$ (b) $\int_R y\,dx$ (c) $\iint_R dx\,dy$ (d) $\iiint_R dx\,dy\,dz$

i) Tangents to the curve at infinity are called________.

(*a*) Cusp (*b*) Node (*c*) Conjugate Point (*d*) Asymptotes

j) The transformation $x = r\cos\theta$, $y = r\sin\theta$ transform the area element $dydx$ into $|J|drd\theta$ where $|J|$ is equal to

(a) r *(b)* r^2 *(c)* $r^2\sin\phi$ *(d)* $r^2\cos\phi$

k) The tangents are real and distinct then the double point is called _____

(*a*) Cusp (*b*) Node (*c*) Conjugate Point (*d*) Asymptotes

l) Which of the following is true?

(a) $\Gamma(m)=2\int_0^\infty e^{-x^2}\cdot x^{2m-1}dx$ (b) $B(m,n) = B(n,,m)$

(c) $\Gamma\left(m+\frac{1}{2}\right) = \frac{2m-1}{2}\frac{2m-3}{2}\ldots\ldots\frac{1}{2}\sqrt{\pi}$ (d) *All are true*

m) Degree of differential Equation $\frac{d^2y}{dx^2} = \left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y^2\right)^{\frac{1}{4}}$ is _____

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 1

n) What is the general solution of $xdy + ydx = 0$?

(*a*) $xy = c$ (*b*) x=cy (*c*) $y = cx$ (*d*) $xyc = 0$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a). Prove that $\int_0^1 x^5(1-x^5)^{10}dx = \frac{1}{3}B(2,11)$. **(05)**

b). Evaluate: $\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx$ **(05)**

c). Show that $B(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ **(04)**

Q-3 **Attempt all questions** **(14)**

a). Show that the series $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots+\frac{1}{n(n+1)}+\cdots\infty$ converges and find its sum. **(05)**

b). Examine the series $\sum_{n=1}^{\infty}\frac{x^n}{n^p}$ for convergence using root test. **(05)**

c). Find radius of convergene for the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$ **(04)**

Q-4 **Attempt all questions** **(14)**

a). Solve: $\frac{dy}{dx} + 2y\tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$. **(05)**

b). Solve: $(x^2 + y^2 + 1)dx - 2xy\, dy = 0$. **(05)**

c). Solve: $\frac{dy}{dx} = cos\, x\, cos\, y - sin\, x\, sin\, y$. **(04)**

Q-5 **Attempt all questions** **(14)**

a). Transform the integral $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2 - y^2}{x^2 + y^2}\, dx\, dy$ by changing to polar Co-ordinates and hence evaluate it. **(05)**

b). Find the area between one arc of the cycloid $x = a(\theta + sin\, \theta)$, $y = a(1 - cos\, \theta)$ and its base. **(05)**

c). Evaluate: $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\, dz\, dy\, dx$ **(04)**

Q-6 **Attempt all questions** **(14)**

a). Using reduction formula Evaluate $\int_0^{\pi} x \sin^7 x \cos^4 x\, dx$ **(05)**

b). Using reduction formula prove that $\int_0^1 x^m (1 - x)^n\, dx = \frac{m!\, n!}{(m + n + 1)!}$ **(05)**

c). Test the convergence of the integral $\int_0^{\infty} \frac{cos\, x}{1 + x^2} dx$ **(04)**

Q-7 **Attempt all questions** **(14)**

a). A circuit containing a resistance R, an inductance L in series is acted on by periodic electromotive force $E\, sin\omega t$. If $i = 0$ at $t = 0$ show that $i(t) = \frac{E}{\sqrt{R^2 + L^2\omega^2}}\left[\sin(\omega t - \phi) + e^{-\frac{Rt}{L}} \sin\phi\right]$, where $\phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$ **(07)**

b). State and prove Legendre's formula. **(07)**

Q-8 **Attempt all questions** **(14)**

a). Find the volume of the solid generated by the revolution of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$ about the $x -$ axis. **(05)**

b). Test the convergence of the integral $\int_{0}^{\infty} \frac{x^2}{\sqrt{x^5 + 1}} dx$ **(05)**

c). Trace the curve $y^2 = \frac{x^2(a - x)}{(a + x)}$; where $a > 0$. **(04)**